Exercise 1 - Computational Models - Spring 2016

Notation: We denote by $\#_\sigma(w)$ the number of times the word $\sigma \in \Sigma^*$ is a substring in the word $w \in \Sigma^*$.

1. For each of the following languages over $\Sigma = \{0, 1\}$, present a drawing representing a DFA that accepts it (correctness proof not needed):

   (a) Σ^*

   (b) $\{11\}||\{0\}^*$

   (c) $\{w \mid w \text{ contains } '10' \text{ and doesn't contain } '001'\}$

   (d) $\{w \mid \#_0(w) \bmod 3 = \#_1(w) \bmod 3\}$

2. For the language $L = \{w01 \mid w \in \Sigma^*\}$ over $\Sigma = \{0, 1\}$ (i.e. the collection of all words ending with 01), present a drawing representing a DFA that accepts it and a formal definition. Prove correctness.

3. Present an NFA (drawing **and** formal definition) and convert it to a DFA (drawing **or** formal definition) for the following languages over $\Sigma = \{0, 1\}$ (correctness proof not needed):

   (a) $1^*(0011^*)^*$

   (b) $\{xy \mid \#_0(x) \bmod 2 = 0 \text{ and } \#_1(y) \bmod 2 = 1\}$

4. Present a regular expression for the following languages over $\Sigma = \{0, 1\}$:

   (a) $\{w \mid w \text{ ends with } 10\}$

   (b) $\{w \mid \#_0(w) \bmod 2 = 0\}$

5. The following question deals with the equivalence between the two definitions for a DFA accepting a string given in class

   Definition 1 *$M = (Q, \Sigma, \delta, q_0, F)$ accepts $w \in \Sigma^*$ if $\widehat{\delta}_M(q_0, w) \in F$.*

Definition 2 *$M = (Q, \Sigma, \delta, q_0, F)$ accepts $w = w_1w_2 \dots w_n$, if $\exists r_0, \dots, r_n \in Q$ s.t.,*

- $r_0 = q_0$.
- $\delta(r_i, w_{i+1}) = r_{i+1}$, *for all* $0 \le i < n$.
- $r_n \in F$.

Prove that if a DFA accepts a string according to definition 1 then it also does so according to definition 2

6. Given that L is a regular language over some alphabet Σ, prove that the following languages are regular:
   (a) $\{xy \mid x \in L,\ y \notin L\}$
   (b) $\{xy \mid (x \in L)\ XOR\ (y \in L)\}$ (i.e. either $x \in L$ and $y \notin L$ or $x \notin L$ and $y \in L$)
7. Prove that the following languages are not regular.
   (a) $L_1 = \{abc \mid a, b, c \in \Sigma^* \ \wedge |ab| = |c| \wedge\ \#_0(a) = \#_0(b)\}$ over $\Sigma = \{0, 1\}$
   (b) $L_2 = \{w \mid \#_a(w) \ge \#_b(w)\}$ over $\Sigma = \{a, b, c\}$
8. Let L be a language such that $\sim_L$ has n equivalence classes (where $n \in \mathbb{N}$). Prove that the minimal DFA accepting L has n states.