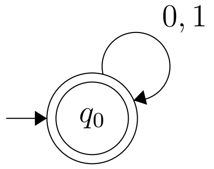
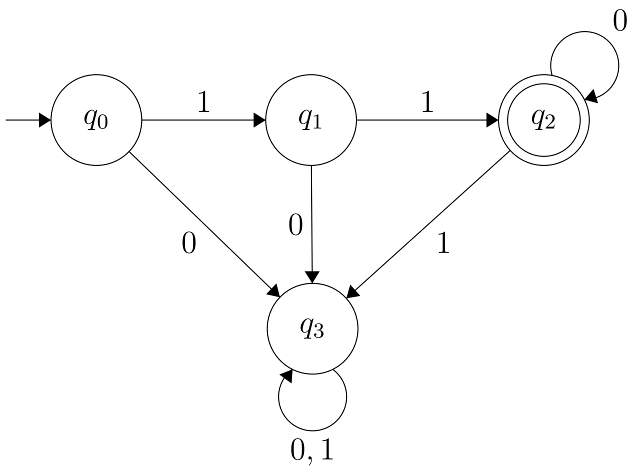
Solution sketch 1 - Computational Models - Spring 2016

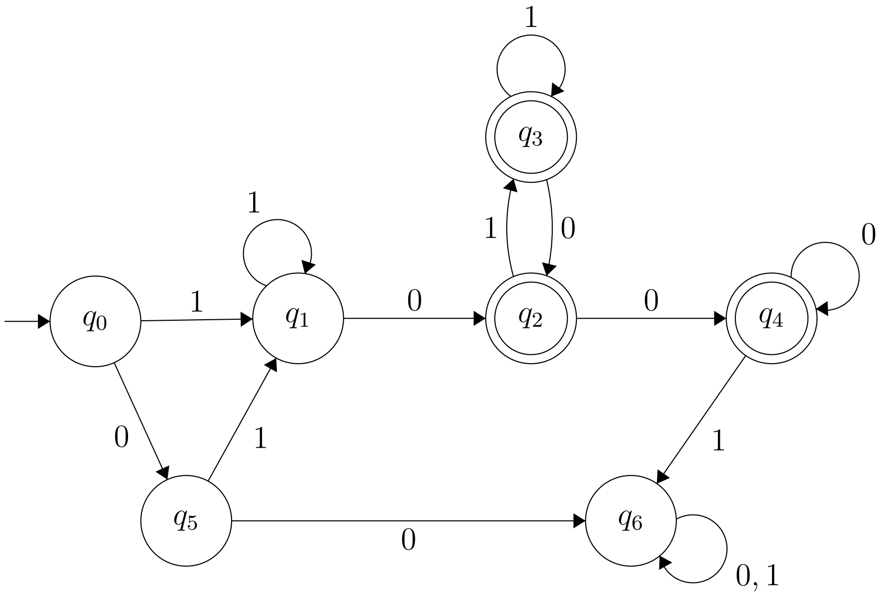
1. (a) Σ^*



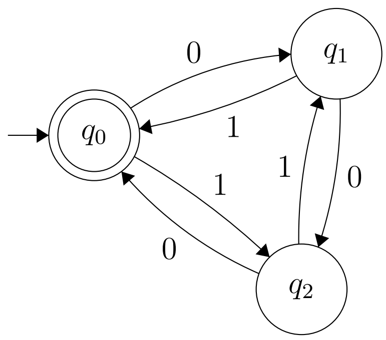
(b) $\{11\}||\{0\}^*$



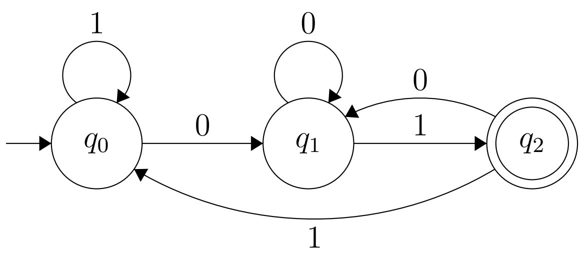
(c) $\{w \mid w \text{ contains } '10' \text{ and doesn't contain } '001'\}$



(d) $\{w \mid \#_0(w) \bmod 3 = \#_1(w) \bmod 3\}$



2. $L = \{w01 \mid w \in \Sigma^*\}$



$A = (Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \delta, q_0, \{q_2\})$, δ :

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_0

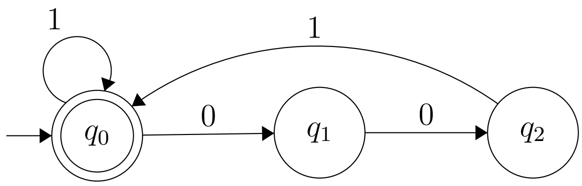
To prove that $\mathcal{L}(A) = L$, you need to prove by indiction on word length the following claim (as seen in class):

Claim. *Let* $\mathcal{L}'_i = \{x \in \Sigma^* \mid \widehat{\delta}(q_0, x) = q_i\}$ *for every* $i \in \{0, 1, 2\}$ *and let*

- $\mathcal{L}_0 = \{w11 \mid w \in \Sigma^*\} \cup \{\varepsilon, 1\}$
- $\mathcal{L}_1 = \{w0 \mid w \in \Sigma^*\}$
- $\mathcal{L}_2 = \{w01 \mid w \in \Sigma^*\}$

Then $\mathcal{L}'_i = \mathcal{L}_i$ *for every* $i \in \{0, 1, 2\}$

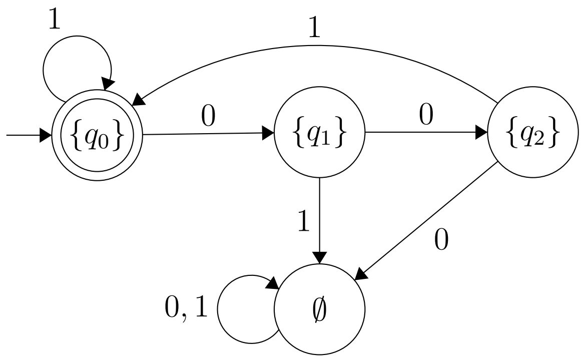
3. (a) $1^*(0011^*)^*$



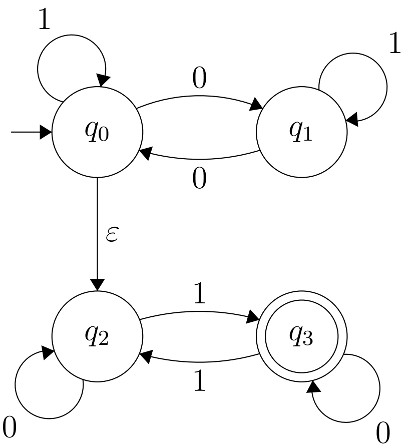
$N = (Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \delta, \{q_0\}, \{q_0\}), \delta$:

	0	1	ε
q_0	$\{q_1\}$	$\{q_0\}$	$\emptyset$
q_1	$\{q_2\}$	$\emptyset$	$\emptyset$
q_2	$\emptyset$	$\{q_0\}$	$\emptyset$

An equivalent DFA



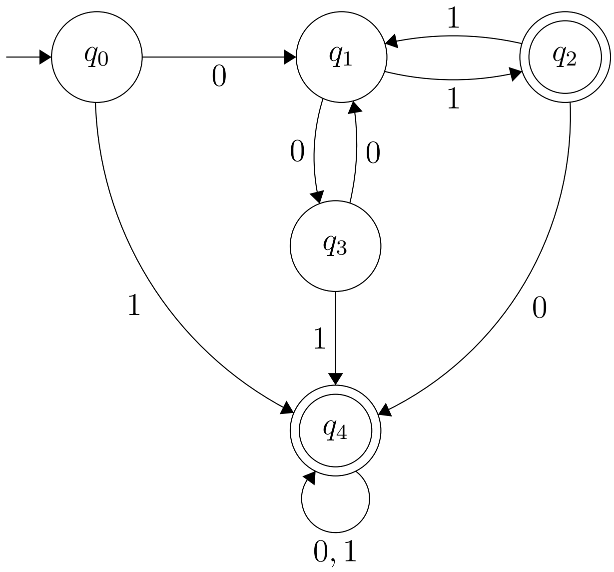
(b) $\{xy \mid \#_0(x) \bmod 2 = 0 \text{ and } \#_1(y) \bmod 2 = 1\}$



$N = (Q = \{q_0, q_1, q_2, q_3\}, \Sigma = \{0, 1\}, \delta, \{q_0\}, \{q_3\}),\ \delta :$

	0	1	ε
q_0	$\{q_1\}$	$\{q_0\}$	$\{q_2\}$
q_1	$\{q_0\}$	$\{q_1\}$	$\emptyset$
q_2	$\{q_2\}$	$\{q_3\}$	$\emptyset$
q_3	$\{q_3\}$	$\{q_2\}$	$\emptyset$

An equivalent DFA (after simplifications)



4. (a) $(0 \cup 1)^*10$
   (b) $1^*(01^*01^*)^*$

5. Can be proved using the claim:

   Claim. *For any* $w \in \Sigma^*$ *s.t.* $w = w_1 w_2 \ldots w_n$ *it holds that* $\widehat{\delta}(q_0, w) = q \Rightarrow \exists r_0, \ldots, r_n \in Q$ *s.t.,*

- $r_0 = q_0$.
- $\delta(r_i, w_{i+1}) = r_{i+1}$, *for all* $0 \leq i < n$.
- $r_n = q$.

The above claim can proved by indiction on word length.

6. (a) This language is $L\bar{L}$. Since L is regular and the regular languages are closed under the operators complement and concatenation, the language at hand is also regular.

   (b) This language is $L\bar{L} \cup \bar{L}L$. Since L is regular and the regular languages are closed under the operators complement, union and concatenation, the language at hand is also regular.

7. (a) Can be proved by the pumping lemma. For any p choose $w = 1^p01^p01^{2p+2}$. Thus $w \in L_1$ and $|w| \geq p$. Let $w = xyz$ s.t. $k = |y| > 0$ and $|xy| \leq p$. Choose $i = 2$, $y = 1^k$ and $xy^2z = 1^{p+k}01^p01^{2p+2}$. If k is odd, $|xy^2z|$ is odd, and thus cannot be a concatenation of abc where $|ab| = |c|$. Otherwise the first half of w is $1^{p+k}01^{p-\frac{k}{2}+1}$ and since it contains only one 0 it can't be written as ab s.t. $\#_0(a) = \#_0(b)$.

   (b) Can be proved by Myhill-Nerode Theorem. Note that for every $m_1 \neq m_2$, $b^{m_1} \not\sim_{L_1} b^{m_2}$.

8. Follows from the proof of Myhill-Nerode theorem, given in the lecture. First, we saw that there exists a DFA with n states accepting L. Second, we saw that n is less or equal to the number of equivalence classes of $\sim_M$ for any DFA M accepting L (defined in the lecture). It is easy to see that the number of equivalence classes of $\sim_M$ is less or equal to the number of states of M. Therefore, n is less or equal to the number of states of any DFA M accepting L. Together, we obtain that n is equal to the minimal number of states of a DFA M accepting L.